In the following guide, boldface type signifies items that can be chosen by clicking with the mouse (menu items and buttons, that is). Although a 3-dim drawing window always has a default figure, you will probably want to begin a drawing session with the **File** menu, from which you can retrieve an **Old** figure that has been **Saved**, or else initiate a **New** figure, which can be either a **Polyhedron** or a **Surface**. There is a **Library** of polyhedral examples, or you can specify the dimensions of the following:

A **Box** is defined by its length, width, and height. A box is positioned with one corner at the origin and its edges parallel to the coordinate axes, by the way.

A right **Prism** is defined by means of a *regular* polygonal base and a height. You can specify the lengths of the base edges and how many there are.

A right **Pyramid** also has a *regular* polygonal base. You must specify either the height of the pyramid or its slant height.

Irregular prisms and pyramids can be created via *2-dim Wingeom*, by the way.

A general **Tetrahedron** (triangular pyramid) is defined by the lengths of its six edges. Any edge fails to intersect exactly one other edge, so the dialog box arranges these lengths in three non-adjacent pairs: one triangular face is {*a*,*b*,*c*}, and *d*, *e*, *f* are non-adjacent to *a*, *b*, *c*, respectively. Click **OK** to see the model. If nothing appears, the data is probably impossible.

The **Vertex** dialog box expects a list of three, four, or five regular polygons (with the same edge length) that will fit together around a common vertex. For example, listing 4,3,5,3,0 tells the program to join a square, an equilateral triangle, a regular pentagon, and an equilateral triangle; because their interior angles sum to less than 360 degrees, this is acceptable input. Click **OK** to see the resulting ring of faces, uniquely arranged so that the concurrent edges have coplanar endpoints.

A **globe** is a polyhedral approximation to a spherical surface. You define its borders by giving latitude and longitude extremes, and then choose the resolution of the latitude-longitude network. You are currently constrained by a 234-vertex upper limit.

There is no library of surfaces, but you can specify dimensions for **Cone**, **Cylinder**, **Sphere**, or **Hemisphere**. A negative height for the cone makes it appear point down.

The rest of the **File** menu:

A Saved figure consists of a single DOS file *filename.ge3* that includes three coordinates for each vertex, a list of all the faces, their colors, and added text. When it is retrieved as an **Old** file, it should appear as it did when you saved it.

Before you **Print** hard copy, you can use **Format** to specify how wide you want the printed image to be. The vertical dimension of the image is determined by the *shape* of the window on the screen, which can be adjusted with the mouse. The size of the window on the paper is important in determining the appearance of any text in the figure (whose size was independently set when you typed it). You will have to experiment a bit. You may also specify where on the paper you want the upper left corner of the image to be. The *x*- and *y-offsets* are measured in inches from the left and top edges of the paper, respectively. Finally, you may specify whether you want to have a **frame** drawn around the image – check the box if you want the frame. When you have made all your choices, click **OK**.

Copy puts the contents of the drawing window on the clipboard, where it can be pasted into any document that accepts the Windows metafile format. You can also put a **bitmap** on the clipboard; this large file disappears from the clipboard as soon as the 3-dim drawing window is closed. **Image size** refers to the pixel dimensions of the drawing window inside the frame.

The **View menu**:

The default mode is to leave **Hidden** lines hidden. The other options are to **dot** hidden lines or to **show** them all.

The **Convex** item is another on-off item. The painter's algorithm used to draw the figures is occasionally fooled by highly irregular faces, by non-convex figures, or by figures that have had faces removed. Whatever the reason, it can happen that elements that should be hidden are not. In such cases, a slower drawing process is needed, which is what unchecking the Convex item enables.

It is occasionally desirable to hide some or all of the labels, or to mark the vertices with bullets or circles. This can be done by using the **Labels/Individual** dialog. The default is to label all vertices alphabetically. Notice that consecutive labels can be abbreviated by a hyphen; thus K–P means K,L,M,N,O,P. Each click of the **Mark** button changes the status of a single group of vertices, according to the state of the **Label** checkbox and the radio buttons **bullet, circle,** and **nothing**.

Click **Label/Font** to alter the font used to display the labels. When you close this dialog box by clicking **OK**, the figure is re-labeled with the new font. Click **Color** to change the color of the labels, and check **Transparent** if you want them not to hide what is underneath.

Click **Labels/Letters** (or press *Ctrl+L*) to hide (or show) all the labels. From time to time, it is necessary to display the alphabetic labels, which the program usually turns on and off automatically; this switch can be used to keep them on.

Click **Offset** to move all labels away from their vertices.

To bring all vertex labels back to their default positions (centered on the points) click **Center**.

Click **Dot** mode (or press *Ctrl+D*) to change the icon used to mark the vertices (nothing, filled circle, open circle). If the labels are centered on the vertices, you will not see any change. Click **Bullet size** to change the size of the icon.

To change the alphabetic labels, put the mouse into text mode (see **Other** below) and right-click the labels that need altering. Or, you can **Swap** the labels for two vertices.

Zoom in to make the figure seem larger in the frame; click **Zoom out** to make the figure seem smaller. (or press *Ctrl-S* or *Ctrl-E* on the keyboard.) The applied dilation factor can be set by clicking **Factor**. The default value is 1.05, meaning that the relative linear dimensions of the frame are increased or decreased by 5 percent. If the factor is less than 1.00, the meanings of *enlarge* and *shrink* will be reversed.

If the figure is not centered in the frame properly, adjust the focus by **Move Center**.

It is also possible to distort figures by applying a **Vertical stretch** within the viewframe. The same **Factor** is used.

To return the figure to its default size and shape, click **Restore**. The program will revert to its usual practice of trying to make every figure fill the viewframe.

Each polyhedral figure is stored as a set of points in 3-dimensional space. To see the underlying system of coordinate axes, click **Axes**. These persist until you click **Axes** again to turn them off. When the axes are on the screen and hidden lines do not show, it takes much longer to draw the diagram.

The appearance of the figure is also affected by moving the **Observer**. This changes the viewpoint and the perspective (but does not affect the coordinates of the points). The closer the observer is to the figure, the closer the vanishing points (the program draws the figure by centrally projecting it onto a viewplane). Click **Turn** (or press *F9*) to move the observer a certain **Angular** distance around the *z*-axis; click **Back** (or press *Ctrl+F9*) to go in the other direction. Click **Up** (or press *F8*) or **Down** (or press *Ctrl+F8*) to change the *z*-coordinate of the observer. Click **In**, **Out**, or **Far out** to move the observer nearer or farther from the *z*-axis. To specify the precise location of the observer, enter **Coordinates**.

In the **Measurement** dialog box, you can make almost any request for numerical data – as long as it can be typed into the edit box, which accepts up to 60 characters. The conventions are illustrated by the following examples, which can be entered in lower case:

Type *AB* and press *Enter* to see the length of segment *AB*.

Type <*ABC* and press *Enter* to see the size (in degrees) of angle *ABC*.

Type *[sin](<ABC)* and press *Enter* to see the sine of angle *ABC*.

Type *ABC* and press *Enter* to see the area of triangle *ABC*.

Type *AB/AC* and press *Enter* to see the ratio of lengths of segments *AB* and *AC*.

Type *ABC|PQR* and press *Enter* to see the size of one of the dihedral angles formed by planes *ABC* and *PQR*.

The **Measurement** dialog box can not be left on the screen while drawing is taking place, so each new calculation is placed in the drawing window. If you do not want a particular item to appear in the drawing, select it (click it) and then click **Hide**. Click **Show** to reverse this process. (Displayed measurements can be moved around like other text items, by the way; just put the mouse in text mode.) To permanently remove a measurement from the master list, select it and click the **Delete** button. To recover this measurement, you will need to retype it. Multiple selections are possible. To insert new measurements into the list, the cursor must be blinking in the edit box. When an item is first entered, it is automatically selected. To copy an expression back into the input box, select it and then click the **Copy** button. As shown above, the program will understand requests for standard functions, provided that the function names are placed inside square brackets. To insert π = 3.14159... into the edit box, you can either type [*pi*] or press *F1*. To remove measurements from the measurement list, highlight them and click the **Delete** button. To insert new measurements into the list, the cursor must be blinking in the edit box. When an item is first entered, it is automatically highlighted. To insert π = 3.14159... into the edit box, you can either type [*pi*] or press *F1*. Press *F2* for a square root sign, *F3* for the exponent ^{2}, *F5* for the golden ratio, and *F6* to ask for the perimeter of a polygon, as in $\sum(ABCDE)$. The program also recognizes the functions

sin, cos, tan, csc, sec, cot, arcsin, arccos, arctan, sqr, log, ln, exp, int, frac, sgn, pi.

As the example above shows, it is necessary to enclose the function name in square brackets, so that the program does not try to read the letters as the name of a polygon.

The **Edit** menu provides several ways of modifying the figure:

You can **Turn** the figure **One Step** (or press *Ctrl+O*), which is defined by the current size of **Angle**. If you click **Many** steps (or press *Ctrl+T*), the resulting animation will not stop until you press a suitable key. Some keys do not halt the process – instead, they cause changes in the rotation data on the fly. The *spacebar* reverses the rotation direction, and the *x*-, *y*-, and *z-keys* send the rotation axis through the centroid of the figure, parallel to the indicated axis.

Occasionally, an unfortunate choice of rotation axis will send the figure crashing into the sides of the frame. The program will simply draw only those parts that fit inside the frame. In such a case, it is probably advisable to temporarily stop the turning and press *Ctrl+E* a couple of times to shrink the diagram.

Although there is always a default axis of rotation, it is sometimes necessary to define a precise rotation axis. Click **Axis** to open the necessary dialog box. The definition is accomplished by typing certain labels into the edit box, then clicking a radio button to tell the program how to interpret the labels:

> The first interpretation is that the labels define a polygonal **Face** of the figure, and that the rotation axis is perpendicular to this polygon and goes through its circumcenter.
>
> The next interpretation is that the rotation axis is defined by two labelled **Vertices**.
>
> The third interpretation is that the rotation axis joins the circumcenter of four labelled vertices to the **midpoint** of the first two.
>
> The fourth interpretation is that the rotation axis joins the circumcenter of four labelled vertices to the **first vertex** in the list.
>
> The next three interpretations are that the rotation axis goes through the centroid of the figure, parallel to the **x-axis**, the **y-axis**, or the **z-axis**. There is no need to type labels into the edit box for this option. This option can even be set *on the fly* (below).

When the rotation data is correct, click **OK** to register your choices. No rotation takes place, however, until you click **Turn/One** or **Turn/Many**.

Another way of modifying the figure is to add **Surfaces** to it. The current choices are **Cones**, **Cylinders**, **Spheres**, and **Disks**. Each such addition is defined in terms of the elements of the figure. For example, a cone must be defined using labeled points for the center of the base circle and the vertex, which must be entered in this order (the *axis* of the cone). Notice that entering *AB* as the radius (instead of a pure number) will enable to program to adjust this definition if the positions of either *A* or *B* change. For each surface, you can elect to **color** some of its parts or to designate them as *transparent*.

Another way to modify the figure is to **Cut** it. This requires that a **Cutting plane** be defined, which can be done by entering

> three **Vertices**;

an **Equation** of the form *ax+by+cz=d* (the default equation belongs to the previously selected plane); or

three points **Marked** on three edges of the figure; a single coordinate is needed for each, with 0 and 1 signifying the first and second vertices of the listed segment, respectively.

When you make a definition and click **OK**, the program displays the intersection of the cutting plane with the faces of the figure, and it also shows the two sections produced by the cut, naming one *Model 1* and the other *Model 2*. To make either of these halves the current figure, click either **Take 1** or **Take 2** in the **Cut** submenu. To recover the last figure that was sliced, click **Restore**.

Another way to modify the figure is to **Join** two figures, which appear in the **Model 1** and **Model 2** windows. These can also be filled with external Saved files. Until they each contain a figure, neither of the following combinations is possible:

To **Merge** the two models means simply to form the figure that is the aggregate of their vertices and faces, identifying any vertices that are within a certain small tolerance of each other. (The tolerance can be altered – click **Tolerance** in the Edit menu.) This is similar to the Paste procedure described next, except that neither figure is turned to force such identifications.

To **Paste** the two Models together means to *match* a face of one with a face of the other. The two specified faces must of course be congruent – within a suitably small tolerance – so that one figure (the second one) can be moved in space to force an identification of corresponding vertices. If the discrepancies between the two faces exceed the tolerance, the process is aborted. (The tolerance can be altered – click **Tolerance** in the Edit menu.) It is possible for the requested identification to force one of the two models to lie *inside* the other – in other words, matching face *ABC* with face *PQR* is quite different from matching face *ABC* with face *RQP*.

The **Faces** dialog allows you to remove faces from a figure, add them back, color them, or subdivide them. You can step through the list by clicking the **face** button, or you can enter the vertices of a polygonal face (or polygonal hole) in the figure instead. You can add or remove edges this way, too. Click the desired button when the correct description is in the edit box. Click **labels** to see (or hide) the labels for the vertices. **Central subdivision** is accomplished by labeling a new vertex – the centroid of the face – then using it to triangulate the face. **Midpoint subdivision** of a polygon is accomplished by joining the midpoints of successive edges. Click the **color** button to display a dialog box that shows the color of the current face; to change the color, just click the desired color in the box. The '**X**' stands for a *clear* face.

Add **altitudes** to the figure by entering a point and either a plane or a segment. To remove such a segment, use the **Faces** dialog described above.

The figure can be moved in relation to the coordinate axes by applying an **Offset**. The result may not be noticeable on the screen unless the axes are showing, or unless the automatic centering mode has been turned off.

To edit **Vertices**, you can modify their **coordinates**. Type the name of the desired point in the edit box, then click **see** to display its current coordinates, or click **move** to change these coordinates to the ones you have just typed into the three edit boxes. The figure will change as a result. To keep the changes made in this way, close the box with the **ok keep** button. To reject all such changes, click **restore**.

New vertices can be defined by means of three space coordinates or by placing a point on an existing line, which only requires one coordinate. Vertices can of course be **deleted**, too. This automatically deletes any faces or edges that were incident there.

The **Other** menu:

The **Fonts** used to display labels and other text can be changed. The available choices are determined by your Windows setup. As soon as you confirm your choices by clicking **OK**, the figure is redrawn.

Click **Euler** to see a count of the *vertices*, the *edges*, and the *faces* for the displayed figure.

When the **Text mode** item is checked, a right mouse button click in the drawing window opens a dialog box that allows you to enter up to 60 characters of text and select a display color for it. If you point at existing text, this text is reopened for editing. Remember that selected text will always disappear if you just start typing *new* text. If you want to edit, then use an editing key *first*.

The left mouse button is for repositioning text, which is hard to place perfectly during the initial insertion. Point at any inserted text in the figure, hold down the left button, and drag the text to its desired position.

The text is not attached to any part of the figure, as you can see by **Turning** the figure while text is on the screen.

The **volume** of the current figure can be requested, though it may be a meaningless value. The program computes the combined volume of pyramids that are formed by joining the faces of the figure to its centroid. If the figure is not convex, it is not easy to interpret the result.

You can change the number of decimal **places** that are displayed in the **Measurement** window.

Click **white faces** to change all the colors in the diagram to white, or **clear faces** to change them all to transparent. There is no undoing this step.

One way to control the **rotation speed** when *Ctrl+T* is pressed is to change the size of the incremental angle. Another way is to apply the brakes by entering a suitable positive number into this dialog box.

The font used for **Measurements** can be changed. A **screen font** will probably not print properly, and it does not scale well, so you may want to choose a **True-Type** font instead. A good choice is MS Line Draw, if it is available. Click **Home** to return all measurements on the screen to the upper left corner of the screen. Check **Units** if you want to see a description of the measurement.

The **PicTeX** item converts the current figure into a text file that can be imported into a TeX document. Enter the width of the desired figure and the box number that it will have in the document. The vertical dimension is determined by the shape of the window.

Here is how you can construct an image of a regular dodecahedron using the preceding features:

1. Start with **File/New/Polyhedron/Vertex**, of type 5,5,5.
2. **Save** the result as Fig1.
3. **Edit/Turn** this figure **One step** of 144 degrees about the face *EHIJA*.
4. The result should be **Saved as** Fig2 (not as Fig1!).
5. **Edit/Join/Merge** Fig1 and Fig2.
6. The result should be **Saved as** Fig3.
7. **Turn** Fig3 about the axis *FN*, just **One step** of 120 degrees.
8. The result should be **Saved as** Fig4.
9. **Edit/Turn** Fig3 One more step (120 degrees).
10. The result should be **Saved as** Fig5.
11. **Edit/Join/Merge** Fig3 and Fig4.
12. The result should be **Saved as** Fig6.
13. **Edit/Join/Merge** Fig5 and Fig6.

This is how the **Library** dodecahedron was produced.